FUZZY MULTI-OBJECTIVE PERIODIC REVIEW INVENTORY PROBLEM IN A DYADIC SUPPLY CHAIN SYSTEM

Dicky Fatrias and Yoshiaki Shimizu
Department of Mechanical Engineering, Toyohashi University of Technology
Toyohashi 441-8580, Japan
Email: dicky@ise.me.tut.ac.jp, shimizu@me.tut.ac.jp

ABSTRACT

This paper presents a fuzzy periodic review inventory model in a dyadic supply chain by incorporating some uncertain parameters. To cope with such uncertainty, a fuzzy multi-objective approach is introduced. A solution procedure is proposed such that several fuzzy goals are satisfied and, in the process, the optimal ordering policy and inventory level are determined.Through providing hypothetically constructed case problem, the usefulness of our proposed model is demonstrated.

KEY WORDS: fuzzy multi-objective problem, inventory, dyadic supply chain

1. INTRODUCTION

The issue of considering uncertainty in inventory problem has received a great deal of attention in the field of production/inventory management. In the context of periodic review system several researches have studies such issue using stochastic approach under different concerns.Recent researches carried out in this direction include Song & Lau (2004), Bijvank& Johansen (2012), Prasertwattana & Shimizu. (2007) and Fatrias& Shimizu (2010).With the development of fuzzy set theory (FST), the fuzzy approach is also employed for the modeling of uncertain parameters inventory problems. The FST copes with the uncertainty related to unavailability and incompleteness of data as well as and imprecise nature of goals of which the use conventional probability distribution impossible in this case.

In this regard,this research proposes a fuzzy multi-objective periodic review inventory model in a typical Supply chain (SC) system in whichsingle-manufacturer, single-retailer is considered. Specifically, we attempt to develop a fuzzy periodic review inventory model in a mixed imprecise and/or uncertain environment by incorporating the fuzziness of demand, lead time and cost parameters.
To cope with such problem, solution procedure is proposed such that several fuzzy goals are satisfied and, in the process, the optimal ordering policy and safety stock of manufacturer and the optimal target stock level of retailer are determined. Through providing hypothetically constructed case problem, we provide the acceptable solutions.

2. SYSTEM DESCRIPTION

In what follows, the proposed inventory model will be described briefly. In all cases, we put the following assumptions.

1. The manufacturer uses the periodic review with lot sizing policy andsafety stock to control its inventory.
2. The retailer uses the periodic review with target stock levelto control its inventory.
3. Only a single product is considered in the model. Without loss of the generality, the manufacturer uses one unit of raw material to produce one unit of product.
4. For both manufacturer and retailer, only one order is allowed to place at any period.
5. Production rate of the manufacturer is assumed fixed and higher than the mean demands.
6. Unfulfilled demand at manufacturer is considered as backorder while unfulfilled demand atretailer is considered as shortages.

The system model is described based on the followingnotation listed for major parameters.

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T = Number of planning horizon.
t = Period(t = 1,2,...,T).
t_p = Number of days in each period.

Parameters of Manufacturer

$\tilde{D}_t$ = Forecast demand of manufacturerat period$t$.
Q_t = Order quantity of manufacturer at period$t$.
PR = Production rate of Manufacturer.
$l\tilde{m}_t$ = lead time of raw material delivery fromsupplier at period t.
Qpr_t = Production quantity produced at period$t$.
Es_t = Ending stock of raw materials of at period t.
Ess_t = Ending safety stock of at period t.
Qm_t = Ordering quantity at period t.
Qb_t = Backorder quantity at period t.
Qsl_t = Sales volume at period t.
EI_t = Ending inventory at period t.
BR = Backorder rate of manufacturer.

Parameters of Retailer

$\tilde{d}_t$ = End customer demand at period t.
$l\tilde{r}_t$ = Lead time of product delivery from manufacturerat period t.
Ir_t = Ending inventory of finished product at period t.
Qsr_t = Shortage quantity after receiving replenishment at period t.
Qor_t = Order quantity at period t.
Qre_t = Replenishment quantity received at period$t$.
LR = Loss rate of retailer.

Cost Parameters

$\tilde{o}_t$ = Order cost of manufacturer at period t.
$\tilde{r}$ = Unit purchasing cost of manufacturer.
$\tilde{m}$ = Unit production cost of manufacturer.
$\tilde{h}$ = Unit holding cost of raw material of manufacturer.
$\tilde{f}$ = Unit holding cost of product of manufacturer.
$\tilde{b}$ = Unit backorder cost ofmanufacturer.
$\tilde{t}$ = Unit transportation cost of manufacturer.
$\tilde{p}$ = Unit purchasing cost of retailer.
$\tilde{c}$ = Unit holding cost of finished product of retailer.
$\tilde{s}$ = Unit shortage cost of finished product of retailer.
TCm = Total Cost of manufacturer.
TCr = Total cost of retailer.

Decision Variables

LS = Lot sizing policy of manufacturer.
ss = Safety stock level of manufacturer.
S = Target stock level of retailer.

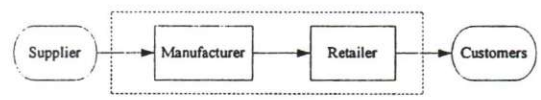

Figure 1. System Configuration

The members in this chain consist of one supplier, one manufacturer, one retailer, and end customers as shown in Figure 1. However, this study focuses on a dyadic relationship in the chain between the manufacturer and the retailer(the supplier and endcustomers are considered as external members in the chain).We assume that these two members are owned and controlled by a central company. Inventoryof each memberis controlled by a periodic review in make-to-stock environment, in whichdemand and leadtime, and cost parameters are considered as a fuzzy number.The inventory level of manufacturer and retailer are reviewed at every period time t, over totally T periods (planning horizon). Each period consists of interval of time t_p days.

2.1 Manufacturer

Manufacturer receives raw materials from outside supplier which has unlimited capacity, transforms it to finished product and then distributes the products to retailer. However, the supplier may delay the supply of raw materials to the manufacturer. Therefore, the manufacturer has to select the appropriate material ordering policy and hold safety stock of product to cope with the uncertainty in demand and delivery lead-time.

The ordering quantity of manufacturer is directly influenced by lot sizing policy (LS) which is adopted for ordering raw material.

After the best pattern of LS is selected, the manufacturer will check the amount of inventory on hand at the beginning of the period. If the amount on hand is less than the sum of the demand and the amount to fill back the safety stock, then the manufacturer will place the order to supplier. Otherwise no order will be issued.

The manufacturer can start the production at the beginning of each period if raw material on hand exists; otherwise the manufacturer has to wait until arrival of raw material from the supplier by time $t+\widetilde{lm}_t$. As a consequence, the production quantity under the combined order condition may become higher than "lot-for-lot" case and results in higher capability to supply retailer's demand (lower shortage cost) at the expense of higher holding cost.

2.2 Retailer

The retailer makes a regular order to the manufacturer periodically to raise up the inventory to the target stock level. The order quantity(Qor_t) is determined by comparing the ending stock level (Ir_t) at the review time t with the desired target stock level (S), which is equal to (S-Ir_t). This target stock level is not only to cover the end customer's demand but also to cover the effect of its fluctuation as well as the late delivery and unfulfilled quantity of products from the manufacturer.

2.3 Objective Functions

This study considers four objective functions to evaluate the system performance. The first objective minimizes total cost of manufacturer(TCm); the second objective minimizes total cost of retailer(TCr); the third objective function minimizesbackorder rate of manufacturer (BR); and thefourth objective function minimizesloss rate of retailer(LR).

$$\text{Min } TCm = \sum_{t=1}^{T}\tilde{o} + \sum_{t=1}^{T}\tilde{u}\times Q_t + \sum_{t=1}^{T}\tilde{h}\times Es_t + \sum_{t=1}^{T}\tilde{c}\times(Ess_t + EI_t) + \sum_{t=1}^{T}\tilde{b}\times Qb_t + \sum_{t=1}^{T}\tilde{t}\times Qsl_t \quad (1)$$

$$\text{Min } TCr = \sum_{t=1}^{T}\tilde{p}\times Qsl_t + \sum_{t=1}^{T}\tilde{c}\times Ir_t + \sum_{t=1}^{T}\tilde{s}\times Qsr_t \quad (2)$$

$$\text{Min } BR_i = \sum_{t=1}^{T}\left(\frac{Qb_i}{Qor_j}\right) \quad (3)$$

$$\text{Min } LR_j = \sum_{t=1}^{T}\left(\frac{Qsr_j}{\tilde{d}_j}\right) \quad (4)$$

3. SOLUTION METHODOLGY

The proposed fuzzy periodic review inventory model is actually a multi-objective mixed integer programming model (MOMIP). To solve the model, a solution procedure is proposed. First, the equivalent crisp MOMIP model is converted into a single-objective MIP model. Then, one evolutionary optimization search method named Differential Evolution (DE) is applied to find an optimal solution.

3.1 The Auxiliary Crisp MOMIP Model

Transforming a fuzzy MOMIP model into an auxiliary crisp MOMIP model require an appropriate method. For this purpose, Jimenez methodis applied because it is computationally efficient to solve a fuzzy problem (See Jimenez et al., 2007).According toJimenez method, the auxiliary eq.: (1)-(4) can be formulated as follows:

$$\text{Min } TCm = \sum_{t=1}^{T}\left(\frac{o^{pes}+2o^{mos}+o^{opt}}{4}\right) + \sum_{t=1}^{T}\left(\frac{u^{pes}+2u^{mos}+u^{opt}}{4}\right)Q_t + \sum_{t=1}^{T}\left(\frac{h^{pes}+2h^{mos}+h^{opt}}{4}\right)Es_t + \sum_{t=1}^{T}\left(\frac{b^{pes}+2b^{mos}+b^{opt}}{4}\right)Qsl_t + \sum_{t=1}^{T}\left(\frac{f^{pes}+2f^{mos}+f^{opt}}{4}\right)(Ess_t + EI_t) + \sum_{t=1}^{T}\left(\frac{t^{pes}+2t^{mos}+t^{opt}}{4}\right)Qsl_t \quad (5)$$

$$\text{Min } TCr = \sum_{t=1}^{T}\left(\frac{p^{pes}+2p^{mos}+p^{opt}}{4}\right)Qsl_t + \sum_{t=1}^{T}\left(\frac{c^{pes}+2c^{mos}+c^{opt}}{4}\right)Ir_t + \sum_{t=1}^{T}\left(\frac{s^{pes}+2s^{mos}+s^{opt}}{4}\right)Qsr_t \quad (6)$$

(1

$$\text{Min } BR_i = \sum_{t=1}^{T}\left(\frac{Qb_i}{Qor_j}\right) \quad (7)$$

$$\text{Min } LR_j = \sum_{t=1}^{T} \left(\frac{Qsr_j}{\left(\frac{t^{pes} + 2t^{mos} + t^{opt}}{4} \right)} \right) \quad (8)$$

3.2 The Proposed Solution approach

The steps of the proposed solution procedures are summarized as follows (Figure 2):

Step 1: Formulate the fuzzy MOMIP (MOMINLP) periodic review inventory model as described in section 2.

Step 2: Determine the appropriate membership function for fuzzy parameters and objective functions. In this formulated problem, fuzzy parameters and objective functions are represented by linear membership function.

Step 3: Convert the fuzzy MOMIP into an auxiliary crisp MOMIP model. To this end, all the imprecise cost parameters in the objective functions as well as the demand and lead time parameters are converted into the crisp ones using Jimenez method.

Step 4: Determine the rage of each objective function by calculating the minimum and maximum value of each of them. To calculate the minimum and maximum value of each objective function, the auxiliary multi-objective crisp model should be solved each time only one objective.

Step 5: Convert the auxiliary crisp MOMIP model into a single-objective MIPbased on Zimmermann'saggregation function (Zimmermann, 1993).The formulation of Zimmermann'saggregation function is as follows:

$$\text{Max } \lambda \quad (9)$$

subject to :

$$\lambda \le \mu_{Zi}$$

$$\lambda, \mu_{Zi} \in [0,1] \quad \forall i$$

where μ_{Zi} is the satisfaction degree of objective function$i$.

Step 6: Solve the corresponding single-objective MIP model.In this stage, the Differential Evolutionalgorithm is appliedto alleviatethe computational complexity. For further information about DE, the reader may refer to Storn and Price, 1997.

Step 7:Output the solution.

4. COMPUTATINAL EXPERIMENT

To illustrate the usefulness of the fuzzy MOMIP model using the proposed solution procedure, a numerical experiment is provided and the result is reported in this section utilizing input parameters shown in Table 1.

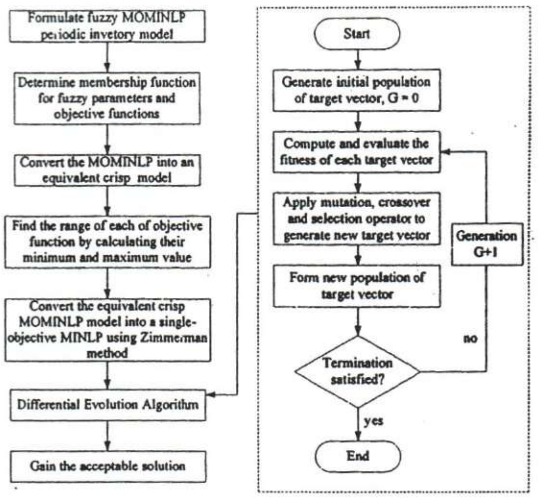

Figure 2. Solution methodology

4.1 Setting the Lower and Upper Bound

For *LS*, the manufacturer has to decide whether to make the order at the beginning of every period or combine the order in a big batch. Therefore, the binary coding is applied to represent the value of *LS*. The*ss*and *S* are considered as the amount of products (units) at the manufacturer and the retailer, respectively. So the integer coding is used to represent these values.

- Lower bound of *LS* is 0,which means"not place the order" in current period but combine it to the previous period's order. Upper bound of *LS* is 1 means "place the order".
- Lower bound of *ss* is set to 0 or no inventory is kept at the manufacturer and during preliminary experiments, the *ss*has never exceeded 20% of the average demand in each period. So, the upper bound of the *ss* is then limited to holding not more than 20% from the average demand in each period.
- Lower bound of *S* should be at least equal to the expected demand during the review timet_p plus average delivery lead time. Based on preliminary experiments, the

upper bound of S has never exceeded 10% of the lower bound.

4.2 Result

The following solution set are obtained based on the proposed solution approach:

$\lambda = 0.525$, $\mu_{Z1} = 0.581$, $\mu_{Z2} = 0.985$, $\mu_{Z1} = 0.525$, $\mu_{Z4} = 0.819$, $Z_1 = \$182{,}551$, $Z_2 = \$81{,}106$, $Z_3 = 17.98\%$, $Z_4 = 17.80\%$.

Table 1. Input Parameters

Input Parameters	
$\widetilde{D}$ (unit)	{561, 751, 1276}[a] {415, 811, 1554} {1411, 1731, 2308} {2376, 3031, 3404} 1505, 1905, 2502} 1452, 1952, 2550}
$\widetilde{d}$ (unit)	{976, 1375, 2076} {1936, 2538, 2836} {2308, 2805, 3104} {804, 1120, 1608} {702, 1012, 1402} {1650, 2054, 2550}
$l\widetilde{m}$ (day)	{1, 2, 5},{2, 3, 6},{1, 3, 5}, {1, 2, 6}, {2, 3, 5}, {1, 3, 6}
$l\widetilde{r}$ (day)	{1, 2, 5}, {1, 2, 6}, {1, 2, 4} {2, 3, 7}, {1, 3, 5}, {1, 3, 6}
t_p	7 days
T	6 periods
PR	250 unit/day
Cost Parameters ($)	
$\widetilde{o}$	{80, 100, 130}
$\widetilde{r}$	{3, 5, 7}
$\widetilde{m}$	{3, 5, 8}
$\widetilde{h}$	{0.11, 0.15, 0.17}
$\widetilde{f}$	{0.10, 0,20, 0.30}
$\widetilde{b}$	{8, 10, 14}
$\widetilde{t}$	{3, 5, 7}
$\widetilde{p}$	{15, 20, 25}
$\widetilde{c}$	{0.10, 0.30, 0.50}
$\widetilde{s}$	{17, 20, 23}
Differential Evolution Parameter	
N_p	50
CR	0.5
F	0.8
G	5,000

[a] {pessimistic value, most likely value, optimistic value}

The maximum value of overall satisfaction degree ($\lambda = 0.581$) is achieved by suggesting the DMs to follow ordering policy in a small lot rather than ordering in a big batch (LS = 100110), and holding a moderate amount of safety stock (ss= 339 unit). Although this decision forces a retailer to hold higher amount of target stock (S = 2322 unit), this decision reduces the total cost of retailer toward its best value. This is indicated by the higher value of achievement level of Z_2 ($\mu_{Z2} = 0.985$) which is close to its best performance.

In addition, from the above solutions, the DMs may define the critical objective and what an appropriate action should be prepared to improve its performance. In this result, the third objective function (minimizing backorder rate of manufacturer) is the critical one because it has the lowest value of satisfaction degree which bounds the value of overall satisfaction degree. To improve its performance, the DMs may reconsider, for example, the ordering policy of raw material – with the expense of other objectives' performance - because it is directly affect the product availability.

5. CONCLUSION

A fuzzy model has been presented in this paper that is useful in determining the best inventory policy in dyadic supply chain system. The uncertainty of the fuzzy parameter is modeled using Jimenez method and the entire formulation is solved by fuzzy multi-objective mix integer programming approach. Through numerical experiment, the usefulness of the proposed solution approach is demonstrated to support decision making process in determining the best inventory policy.

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